Kittel TP

2.3 (a) For single system, $\sigma = \ln g(N, n)$

$$= \ln \frac{(N+n-1)!}{n!\,(N-1)!}$$

using stirling approx, and treat $N-1 \approx N$,

$$\sigma = (N+n)\ln(N+n) - (N+n) - n\ln n + n - N\ln N + N$$

$$= (N+n)\ln(N+n) - n\ln n - N\ln N$$

Davidson Cheng

1.6.2024.

Kittel TP

2.3(b).

$$\sigma = (N+n)\ln(N+n) - n\ln n - N\ln N$$

substitute $n = \frac{U}{\hbar\omega}$

$$\boxed{\sigma(U,N) = \left(N+\frac{U}{\hbar\omega}\right)\ln\left[N+\frac{U}{\hbar\omega}\right] - \frac{U}{\hbar\omega}\ln\frac{U}{\hbar\omega} - N\ln N}$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N$$

$$= \frac{1}{\hbar\omega}\ln\left[N+\frac{U}{\hbar\omega}\right] + \left(N+\frac{U}{\hbar\omega}\right)\left(N+\frac{U}{\hbar\omega}\right)^{-1}\left(\frac{1}{\hbar\omega}\right)$$
$$-\left[\frac{1}{\hbar\omega}\ln\frac{U}{\hbar\omega} + \frac{U}{\hbar\omega}\left(\frac{U}{\hbar\omega}\right)^{-1}\left(\frac{1}{\hbar\omega}\right)\right]$$

$$= \frac{1}{\hbar\omega}\ln\left[N+\frac{U}{\hbar\omega}\right] + \frac{1}{\hbar\omega} - \frac{1}{\hbar\omega}\left[\ln\frac{U}{\hbar\omega} + 1\right]$$

$$\Rightarrow \frac{\hbar\omega}{\tau} = \ln\left[N+\frac{U}{\hbar\omega}\right] + 1 - \ln\frac{U}{\hbar\omega} - 1$$

$$= \ln\left[\frac{N+\frac{U}{\hbar\omega}}{\frac{U}{\hbar\omega}}\right] = \ln\left[\frac{N\hbar\omega}{U} + 1\right]$$

$$\Rightarrow \exp\left[\frac{\hbar\omega}{\tau}\right] - 1 = N\hbar\omega/U$$

$$\boxed{U = \frac{N\hbar\omega}{\exp\left[\frac{\hbar\omega}{\tau}\right] - 1}}$$

Davidson Chen

1.6.2024